critical points and the second is a restriction to long waves. The surface condition leads to $\rho(-(U(0)-c)\phi'(0)+\phi(0)U'(0))=(\rho g+Tk^2+s-mk^2c^2-imdkc)a$. Then, evaluation with the assumed solution produces the approximate dispersion relation $\rho(U_0-c)^2k=\rho g+Tk^2+s-mk^2c^2-imdkc$ where $U_0=U(0)$. Thus unstable c_i's are easily determined for all parameters.

Finally, consider an interesting example for the Couette flow $U(y)=\beta y+\gamma$. In this case $U''=0$ and if $(U-c)$ is nonzero in $(-H,0)$, the solution ϕ to $\phi''-k^2\phi=0$ satisfying Eq. (2) is proportional to $\tanh kH \cosh ky+\sinh ky$. Substitution in Eq. (3) leads to the dispersion relation

$$\frac{\phi'(0)}{\phi(0)}=\frac{(\gamma-c)\beta\rho+\rho g+Tk^2+s-mk^2c^2-imdkc}{\rho(\gamma-c)^2}$$

$$=\frac{k}{\tanh kH}$$

Now this is equivalently $\phi(0)=R^{-1}\phi'(0)$ where R measures an effective rigidity. In the rigid wall limit we find that $\phi(0)=0$ and hence ϕ vanishes identically; the basic flow, therefore, has no eigensolutions of the discrete c-spectrum. Thus we conclude here that it is the allowance of wall flexibility that gives rise to wavelike perturbations.

Discussion and Concluding Remarks

The foregoing results for the inviscid shear flow stability over compliant surfaces were obtained to supplement the more detailed viscous studies of Benjamin[1] and Landahl[2] using the Orr-Sommerfeld equation. Our work focuses essentially on explicitly obtainable eigenvalue bounds, sufficiency conditions, and dispersion relations for the Benjamin-Landahl membrane model, although, for somewhat restrictive parameter ranges.

While the results reported here are not directly applicable to the analysis of finite length plates (panel flutter) per se as contrasted with infinite length membranes, the general methods may be used in conjunction with alternative structural models and with only minor modification. The basic problem, of course, is one for fluid and solid interaction, and a substantial body of more general work appears in the aeroelastic literature. Noteworthy among these are some significant theoretical models for shear flows over flexible boundaries, for example, those pursued by Dowell and his collaborators.[4-6] These studies allow for both finite plate dimensions as well as fluid compressibility; their numerical results, moreover, compare favorably with the experiments of Muhlstein et al.[7,8] The analytical approaches adopted here and in Refs. 4-6 are more or less equivalent. The simpler interaction model examined here, though, leads to simple closed-form results, and these may be useful in various applications.

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Use of Matched Pressure Initial Conditions for Predicting Low-Altitude Rocket Plume Radiation

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Nomenclature

A_{max} = plume maximum cross-sectional area, $A_{max}=9\pi(FD)^{1/2}/16q_\infty$
A = nozzle area
C_D = plume drag coefficient: $C_D=D/q_\infty A_{max}$
C_F = nozzle thrust coefficient:

$$C_F=\left\{\frac{2\gamma_e^2}{\gamma_e-1}\left(\frac{2}{\gamma_e+1}\right)^{(\gamma_e+1)/(\gamma_e-1)}\times\left[1-\left(\frac{p_e}{p_c}\right)^{(\gamma_e-1)/\gamma_e}\right]\right\}^{1/2}+\frac{p_e-p_\infty}{p_c}\frac{A_e}{A^*}$$

$$C_{F_{max}}=\left\{\frac{2\gamma_e^2}{\gamma_e-1}\left(\frac{2}{\gamma_e+1}\right)^{(\gamma_e+1)/(\gamma_e-1)}\times\left[1-\left(\frac{p_\infty}{p_c}\right)^{(\gamma_e-1)/\gamma_e}\right]\right\}^{1/2}$$

D = plume drag
F = missile thrust
M = Mach number
p = pressure
q = dynamic pressure
r = plume or nozzle radius
T = temperature
v = velocity
γ = specific heat ratio

Subscripts

e = nozzle exit
∞ = ambient
c = combustion chamber

Superscript

$*$ = nozzle throat

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I. Introduction

PREDICTIONS of rocket exhaust plume properties at low altitudes (<60-80 km) are required in a wide variety of studies dealing with optical signatures and electromagnetic scattering cross sections. The nozzles are underexpanded and the exhaust flow passes through two distinct regions: a near-field expansion region in which the pressure equilibrates to the ambient level through successive shock and expansion waves; and a subsequent, far-field region where mixing and afterburning with the external air stream occur at essentially constant pressure. A unified and detailed description of this entire plume structure requires elaborate computational tools.

One simplification that is frequently employed ignores the details of the expansion region and treats only the constant pressure mixing. This approach is motivated by the predominant importance of the mixing in determining the afterburning and viscous dissipation, and consequently the plume observables. The averaged plume properties evaluated at the ambient (matched or equilibrated) pressure are used as uniform initial conditions for the subsequent mixing. One specification of these properties ignores the shock losses completely and thus assumes an isentropic expansion. This provides the lowest temperature and smallest plume diameter at the initiation of mixing. A more realistic approach uses a simple integral method that accounts for the nonisentropic expansion by relating it to the overall plume drag. Sukanek[1] has proposed such an analysis, that utilizes the plume drag from the Jarvinen-Hill[2] model of a vacuum plume from a vehicle moving at hypersonic velocity.

Sukanek's analysis, however, contains an inconsistent specification of the plume drag. This Note corrects this inconsistency and presents new expressions for the matched pressure properties. The validity of the corrected analysis is established by a comparison with averaged matched pressure properties obtained from detailed numerical solutions for the near-field expansion which explicitly account for the plume shocks. The limitations of the corrected analysis are identified. In addition, the larger question of the utility of the matched pressure initial conditions is examined for specific application to plume infrared signature studies.

II. Corrected Matched Pressure Properties

The plume drag in the integral momentum balance [Eq. (3) of Ref. 1] is defined in terms of a drag coefficient C_D which uses as a reference area the maximum plume cross-sectional area $A_{\max} = 9\pi(FD)^{1/2}/16q_\infty$. However, Sukanek evaluated the drag as $q_\infty \pi r^2 C_D$, using the pressure equilibrated area, πr^2, rather than $A_{\max}$. Since $\pi r^2 < A_{\max}$, this formulation always predicts too little drag. The correction suggested here simply uses the maximum cross-sectional area, consistent with the defintion of C_D, or more directly, the plume drag itself; $D = F(C_{F_{\max}}/C_F - 1)$ [Eq. (12) of Ref. 1]. The average velocity, temperature, and radius at matched (ambient) pressure are then given by (for a thermally and calorically perfect gas)

$$v/v_e = 1 + (1 - p_\infty/p_e)/\gamma_e M_e^2 + (C_{F_{\max}} - C_F)p_e A^*/\gamma_e p_e M_e^2 A_e \quad (1)$$

$$T/T_e = 1 - \tfrac{1}{2}(\gamma_e - 1)M_e^2[(v/v_e)^2 - 1] \quad (2)$$

$$r/r_e = [(p_e/p_\infty)(T/T_e)/(v/v_e)]^{1/2} \quad (3)$$

Predictions of the average temperature obtained from the corrected analysis are compared in Fig. 1 with those from the original analysis and with isentropic values for a typical large liquid propellant booster (Sukanek's case A). The corrected analysis predicts average temperatures that are uniformly lower than those from Sukanek's original formulation. In order to assess the corrected analysis, we also include in Fig. 1 average temperatures based on predictions using an inviscid plume code (SCIPPY)[3] that accurately accounts for the detailed shock structure. This comparison indicates that the corrected Sukanek analysis does indeed provide correct averaged matched pressure properties. It should be noted that Sukanek also reported a favorable comparison between his original analysis and detailed numerical solutions. However, those numerical results utilized an approximate shock configuration that was found to overpredict the entropy rise.[4] Hence, the initial agreement must be regarded as fortuitous. More recent numerical results[4] are also included in Fig. 1 and are in excellent agreement with those of SCIPPY and the corrected Sukanek analysis.

In spite of the demonstrated agreement with the detailed numerical solutions, there is a potential source of error in the integral analysis as proposed by Sukanek. The Jarvinen-Hill hypersonic drag model predicts a plume drag that is independent of vehicle velocity. The averaged matched pressure plume properties from this analysis, like those obtained from an isentropic expansion, therefore depend only on the ambient pressure. Agreement between the integral analysis and numerical solutions shown in Fig. 1 implies that the drag predictions are essentially the same, at least for the particular vehicle and trajectory examined here. However, this may not be uniformly true for all missiles or trajectories. In particular, for slower vehicles, for which the Jarvinen-Hill model may be less accurate, the true average properties may be different

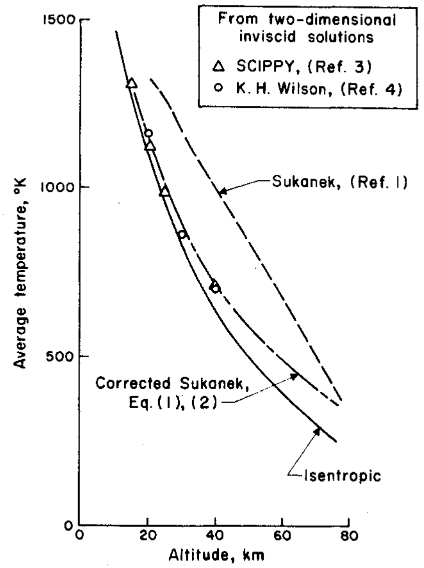

Fig. 1 Average temperature at matched pressure: ($\gamma_e = 1.23$, $p_e = 0.892$ atm, $M_e = 3.06$, $A_e/A^* = 8$, $T_e = 1854$ K, $v_e = 2.78$ km/s, $r_e = 0.775$ m).

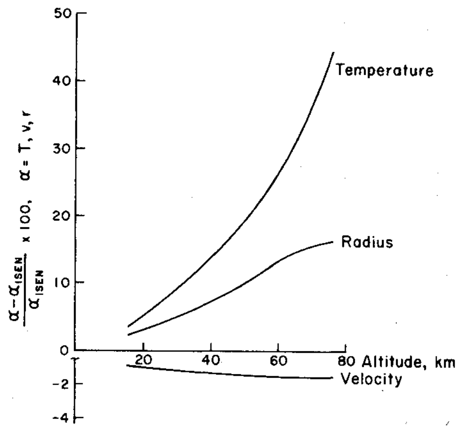

Fig. 2 Normalized average temperature, velocity, and radius.

(higher temperatures, lower velocities, and larger radii than those predicted using the Jarvinen-Hill model). This trajectory independence is due only to the use of the Jarvinen-Hill model and can be rectified by the use of an alternative drag prediction that does account for vehicle velocity.

The averaged pressure-matched velocity, temperature, and plume radius are shown in Fig. 2. They are normalized by the isentropic values in order to make the results less sensitive to the particular case examined here. The differences between the integral and the isentropic properties increase with increasing altitude and are important. For example, infrared radiation scales with r^2 or r^3, for an opaque or optically thin plume, respectively. The spectral emission depends on T^m, $3 \leq m \leq 7$, depending on the wavelength and temperature. Differences of 10% in the initial radius and temperature can, therefore, lead to differences of 60-150% in the radiation.

III. Applicability of Matched Pressure Initial Conditions

The validity of the corrected simple integral solution has been demonstrated in a typical application. However, an issue of even greater importance involves the adequacy of plume mixing predictions which utilize these uniform, average properties as initial conditions. In order to make this assessment for one class of problems, we have calculated the infrared radiation from the plume of the aforementioned large booster at an altitude where shock strengths are appreciable and afterburning is minimal (i.e., $h > 30$-40 km). A sequence of flowfield predictions was made that account for the shock structure in the expansion region in an increasing degree of detail, namely:

1) Constant pressure mixing with uniform, matched pressure initial conditions obtained from an isentropic expansion (this specification ignores the shocks entirely, and is included only to serve as a basis for comparison).

2) Constant pressure mixing with uniform, matched pressure initial conditions obtained from the corrected Sukanek analysis.

3) Constant pressure mixing with nonuniform, matched pressure initial conditions; the actual, two-dimensional distribution obtained from SCIPPY, was employed from which the average properties in case 2 were obtained. This distribution accounts for the nonuniform entropy distribution, i.e., the temperature varies from 1100 K near the axis where shock strengths are maximum to the isentropic value of 630 K at the outer edge. The average temperature of 710 K is essentially the same as that predicted by the corrected integral method (Fig. 1).

4) The mixing was initiated near the nozzle exit and formed a shear layer along the curved plume/air interface. The mixing accounted for longitudinal and transverse pressure gradients and variable edge conditions defined by the inviscid flow. The "overlaid" flowfield was constructed from the superposition of this mixing layer on the detailed inviscid plume expansion.

The mixing/afterburning calculations were performed with the BOAT code[5] using a two-equation turbulence model. The computational aspects of the "overlaid" approach are discussed in Ref. 6. Infrared radiation from the four nonhomogeneous plumes was calculated according to a band model formulation.[7]

Infrared radiation from the entire effective radiating plume length was calculated for the near infrared wavelength region for each flowfield. The radiant intensities are summarized in Table 1 and are normalized by that predicted for case 1 in order to make a relative assessment. The most important observation is that the most realistic flowfield description, case 4, yields a plume signature that is three times larger than that calculated for the matched pressure initial conditions, case 2. Such differences have been previously recognized and estimated for conventional liquid propellant plumes (see Ref. 6). The additional calculation for the nonuniform, pressure equilibrated startline, case 3, shows about a 70% increase over the radiation for the uniform startline, case 2. This suggests that the nonuniform initial conditions for the far-field mixing contribute about half the additional radiation due to the inviscid/shock structure and that the remainder is due to direct and indirect contributions from the near-field wave structure. It is tentatively concluded that for altitudes of approximately 30 km and higher, pressure equilibrated initial conditions will yield total radiation estimates that are significantly low, regardless of the accuracy of the pressure equilibrated initial conditions themselves.

Table 1 Relative plume radiant intensities in near infrared (large liquid propellant booster at altitudes greater than 30-40 km)

Initial conditions for mixing	Relative intensity
1) Uniform, pressure equilibrated, isentropic expansion	1.0
2) Uniform, pressure equilibrated, Eqs. (1-3)	2.2
3) Nonuniform, pressure equilibrated	3.7
4) Mixing overlaid on inviscid/shock pattern	6.6

It is anticipated that the difference between the plume emission using pressure equilibrated initial conditions and that using the more exact overlaid approach will decrease with decreasing altitude. The principal reasons are that as the altitude decreases, shocks diminish in strength, afterburning contributions become increasingly important, and the inertia-dominated inviscid expansion region decreases in size relative to that of the overall radiating plume. Above about 35 km, afterburning is minimal and peak temperatures in the plume are produced almost entirely by shocks and viscous dissipation. A more extensive series of calculations is in progress to establish firmly the conclusions drawn here and will be reported in Ref. 8. A description of the computational flowfield models and overlaid procedure employed here will be available in Ref. 9.

IV. Conclusions and Summary

It is known that uniform, pressure equilibrated initial conditions, using either an isentropic expansion or Sukanek's analysis, are widely used in plume infrared signature and radar cross-section studies. The correction to Sukanek's analysis and its subsequent verification by detailed numerical codes show that correct average matched pressure conditions are predicted by this simple integral analysis. More importantly, however, the specific calculations with infrared radiation as the end result imply that the matched pressure initial conditions have a limited range of utility; ending at some altitude below 30-40 km. The improving capability[3,5,9] to predict accurately and efficiently the detailed gasdynamics of rocket plumes now allows for the relaxation of unverified simplifications, such as the one considered here, which are currently in use.

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Comment on "Wall Shear Stress Measurements in a Shock-Wave Boundary-Layer Interaction"

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MAJOR advances have been made in establishing numerical solutions to turbulent boundary-layer separation problems, yet there are still many serious discrepancies between predicted flow characteristics and experimental results. The acquisition of accurate skin friction measurements in reverse flow regions is of primary importance in evaluating numerical techniques as well as in adding to the basic understanding of separated flow phenomena.

Recent studies by Murthy and Rose[1] provided valuable data concerning the reliability of various skin friction measuring devices. Their attempts to provide correlations of experimental skin friction values with numerical solutions of Baldwin and Rose[2] and Baldwin and MacCormack,[3] however, may have biased their results. The skin friction gages used by Murthy and Rose could not distinguish between positive and negative skin friction values, but they assumed the reverse flow skin friction coefficients to be negative in order to correspond with the negative values predicted by numerical techniques. Plotting the data in this manner resulted in a significant jump between positive and negative values at the separation and reattachment points, as shown in Fig. 1. If the skin friction coefficients in the reverse flow region had instead been plotted as positive values, it could then be assumed that an appreciable error existed in all of the data points.

Considering flow conditions for the test case ($Re_\delta = 0.97 \times 10^6$, $\delta = 1.7$ cm, $T_0 = 270$K, $T_w = 60°$C, and $M_\infty = 2.9$), one can arrive at a unit Reynolds number of 5.6×10^5/cm and $T_w/T_\infty = 3.1$. Using the boundary-layer relationship of Johnson and Kaufman,[4] $\delta/x = 0.3122 (Re_x)^{-0.1622}$, to describe the boundary-layer thickness as a function of the local Reynolds number, based upon distance from the wind tunnel throat x, one can calculate a local Reynolds number at interaction to be 5.6×10^7. The technique of Truitt,[5] after Van Driest,[6] can then establish a local skin friction coefficient of approximately 0.0011 for the undisturbed flow. This is slightly lower than the value recorded by Murthy and Rose (0.0014) but corresponds to the upstream skin friction coefficient measured by Settles et al.[7] at comparable conditions. If one thus assumes an error of approximately 0.0003 in the measurements of Murthy and Rose, their data can then be plotted as shown in Fig. 2.

It is suggested that Fig. 2 might be a more accurate representation of the skin friction measurements of Murthy and Rose, even though it indicates negligible skin friction values in the reverse flow region and implies that significant inaccuracies might exist in the numerical solutions. It is assumed that the rather large fluctuations in the sensor gage

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